eJournal of Tax Research (2008) vol. 6, no. 2, pp. 169–194

Modelling the Effects of Corporate Taxation in the Underground Economy

Konstantinos Eleftheriou*

Abstract
This paper develops a two-sector search model of the labour market in which firms in one sector (the informal sector) evade profit taxes (underground economy). A comparative static analysis is employed to analyze the impact of corporate taxation on unemployment, occupational choice of individuals, mix of jobs, welfare of agents and the size of informal sector. The findings suggest that profit, firing and payroll taxation have the same effects on the above economic variables. However, if a certain condition does not hold, then the number of individuals searching for jobs only in the informal sector decreases with profit taxes. The above result implies that the adoption of active labour market policies, which accelerate the matching process between employers and employees, will mitigate the positive (negative) impact of taxation on the size of underground economy (wages).

1. INTRODUCTION

Many empirical studies indicate that the size of shadow economic activity is growing year by year. More specifically, the increase of the size of the underground economy (measured as a percentage of the GDP) between 1960 and 1995 for the United States was 6%. In other OECD countries such as Germany and Norway growth was even higher [see, for example, Schneider and Enste (2000)]. The negative impact of growing shadow activities on the economy can take many forms. This diversity has motivated researchers to find the main causes for the growth of underground economy and potential remedies. A widely accepted explanation for the increasing size of the underground sector is the existence of high marginal income tax rate [Loayza (1996), Giles (1999a), Thomas (1992) et al.]. Higher taxes increase the incentive for tax evasion and consequently the size of the shadow economy.

Tax evasion as a topic of theoretical investigation was first suggested by Mirrlees (1971). Since then numerous papers have been written about the phenomenon of tax evasion. Most theoretical analyses try to model the phenomenon of tax evasion by using standard portfolio theory of choice and uncertainty. An early example of such a theoretical approach is the work of Allingham and Sandmo (1972). In their model, they examined two cases, one static and one dynamic. In the dynamic case individuals have to make a sequence of tax declaration decisions. They argued that the impact of

* Department of Economics, University of Essex, email <kostasel@otenet.gr>. I am deeply grateful to my PhD thesis supervisor, Dr Eric Smith, for his guidance and support. Also, I would like to thank Professor Melvyn Coles, Dr Miltos Makris and an anonymous referee for their very helpful comments and suggestions. This paper is based on the second chapter of my PhD thesis in University of Essex. Any remaining errors are my own.

the tax rate on the reported income in the static case is ambiguous. However, Yitzhaki (1974) showed that if the fine for tax evaders is imposed on the evaded tax and not on the undeclared income then Allingham and Sandmo model gives a positive relationship between tax rates and tax evasion.

Kesselman (1989) developed an intersectoral model of income tax evasion in order to examine the general equilibrium aspects of it. In his analysis, workers are assumed to have heterogeneous evasion costs and the outputs produced in each sector are imperfect substitutes. He concluded that if an increase in taxation does not affect the level of evasion costs and if the consumption pattern of government is the same with that of households, then the higher the tax, the greater the extent of evasion. However, if one of the above does not hold then we may get the inverse outcome.

While the above theoretical works focus on employee tax evasion, Blakemore, Burgess, Low and Louis (1996) examined employer (corporate) tax evasion. They showed that an increase in the payroll tax rates is likely to increase the employer tax evasion in the UI program. According to their analysis an increase in tax rates has two opposite effects: (i) increases the evasion incentives by increasing the return from incomplete disclosure and (ii) decreases evasion incentives by reducing the optimal size of the workforce (more unemployment). Nevertheless, under certain parameter values, the latter effect is dominated by the former one. Moreover, they empirically verified the above findings.

The main contribution of our paper to the literature on tax evasion is the use of a Pissarides-type matching model of the labour market where workers have heterogeneous productive skills.[1] More specifically, we examine an intersectoral (two sectors) model of the labour market where workers and vacant jobs meet each other according to a constant returns to scale matching function. One of the sectors is characterized as the underground sector in the sense that in this sector filled jobs evade taxes (profit and firing[2] taxes). A rather striking result of our analysis is that under certain conditions, the welfare of a subgroup of the total population of individuals increases as profit tax rises.

In the standard Pissarides model of the labour market with stochastic job matchings, an increase in profit taxation (all firms are taxed with the same tax rate) will result in a decrease in the welfare of individuals. However, the existence of the untaxed sector in our model will initiate certain effects which will lead to a welfare improvement of some individuals. More specifically, when profit and severance tax increase, the following effects occur:

(i) The mix of jobs changes in favour of the underground sector (i.e., underground sector becomes more attractive for firms). As a result of this change, the probability that an unemployed worker will meet a vacancy in the underground sector is greater than the probability that he will meet a vacancy in the taxed sector.

[1] The framework of the model resembles that of Roy (1951), where individuals are assigned a random vector the elements of which indicate their productivities in each sector.

[2] Firing taxes (severance payments) are the payments – benefits received by an employee when he is laid off (e.g,. layoff compensation).

(ii) The flow of firms out of the taxed sector is not equal to the flow of firms into the tax evading sector and therefore the total number and the arrival rate of vacant jobs decrease. In a Nash bargaining process the former effect increases the outside option (threatening point) of individuals who search only for jobs in the underground sector whereas the latter effect decreases it. If the impact of (i) is greater than that of (ii) on the outside option of those individuals then their welfare unambiguously increases.

Apart from welfare effects, we investigate the impact of an increase of profit and firing taxes on total unemployment and relative sectoral employment (thus measuring the size of the underground sector). We also examine how profit and severance taxes influence the occupational choice of individuals and the mix of vacancies. We find that payroll taxation in Albrecht et al (2006) work has the same impact on the above economic variables with the corporate and the firing tax in our analysis. However, our assumption about the endogeneity of the arrival rate of informal sector jobs is the driving force behind the result, that less people accept only informal sector jobs as corporate income tax or severance tax increase, when the parameter which captures the "technological" advances in the matching process is high enough. This result is the opposite from that of the firing and payroll tax in Albrecht et al (2006). Such a result cannot be obtained in the case of payroll taxation in the Albrecht et al paper, since the arrival rate of informal sector jobs is exogenous. This result also leads us to important policy implications. Active labour market policies favouring technological advances in the matching process between employers and employees (technological advances in the matching process include reforms such as the computerization of employment offices, job advertising on the internet, job-search assistance policies, governmental subsidies into policies helping the matching process etc.), will 'moderate' the expansion of the underground sector caused by an increase in profit/firing taxes. Moreover, the adoption of such policies, will limit the reduction of wages induced by higher taxes (corporate/firing).

The model which is closest to ours is that of Albrecht, Navarro and Vroman (2006). In their paper, they extended the standard Mortensen and Pissarides (1994) model of the endogenous job destruction by adding an informal sector and allowing for worker heterogeneity. More specifically, they assumed that the arrival[3] and the destruction rate of informal sector jobs are exogenous whereas formal sector jobs are characterized by endogenous (arrival and destruction) rates. The assumption of the exogenous arrival rate of informal sector jobs ignores the inter-relationship between the two sectors (formal-informal). More specifically, the number of vacancies (vacancy creation-job destruction) and the total level of unemployment in the formal sector will have an impact on the arrival rate of jobs in the informal sector and vice versa. The failure of their model to incorporate the aforementioned interaction will 'eliminate' the efficiency of active labour market policies in 'smoothing' the impact of taxation on the size of the underground economy. Moreover, Albrecht et al assumed that all individuals who are employed in the informal sector earn the same income (homogeneous productive abilities) while the workers in the formal sector have different productivities and hence they are paid different wages. This homogeneity assumption may have a drawback, since if the income received is too small –close to

[3] The assumption of the exogenous arrival rate implies that the average unemployment rate among those who accept only informal sector jobs remains the same as payroll tax increases.

zero– then nobody will work for the informal sector. Moreover, it can be considered as a bit unrealistic. The homogeneity assumption regarding the earnings in the informal sector again limits the effectiveness of active labour market policies in the reduction of the distortionary effects of taxation on wages. In our analysis, individuals are heterogeneous regarding their productive abilities in both sectors.[4]. Furthermore, we assume that there are no productivity shocks (i.e., jobs in both sectors are characterized by the same exogenous destruction rate) and that all jobs arrive at the same endogenous rate.

The paper is organized as follows. In the next section, the basic model is presented. Section 3 examines the nature of equilibrium. Section 4 presents the comparative statics analysis and simulates the model. Section 5 concludes.

2. THE MODEL

2.1 ENVIRONMENT

We consider a continuous-time model with risk neutral and infinitely lived agents. A continuum of workers, normalized to unity, participate in the market. There are two sectors: 1 and 2. Before entering into the labour market, each individual is endowed with a two-dimensional skill vector $a \equiv (a_1, a_2)$, where a_1 and a_2 are independent random variables and uniformly distributed over the interval [0,1]. Denote the density function $f(a_1, a_2) = 1$.[5] Each element of this vector indicates the productive capability that an individual has in the corresponding sector. Hence, the output produced by an individual with skill vector a is a_1, if he is employed in sector 1, and a_2, if he is employed in sector 2.

Workers are either employed or unemployed, and jobs are either filled or vacant. Each job can employ only one worker and vice versa (i.e., workers can be employed either in sector 1 or in sector 2 but not in both sectors). Filled jobs 'die' at an exogenous rate δ. Assume free entry for vacancies, i.e., vacancies are created whenever it is profitable to do so (the long-run nature of the model allows the assumption of the free entry). Firms and workers discount the future at the same rate r. The cost of holding a vacancy is constant and equal to c. The value of the unemployment insurance benefit is equal to b. For simplicity, we will assume that b is equal to zero. Firms in sector 1 pay a profit tax at rate τ (where $0 \le \tau \le 1$) in each period. Sector 2 is assumed to be the underground sector of the economy and therefore no tax is paid by sector 2 firms. However, profit tax evasion has an explicit cost. We will assume that the probability of being caught evading taxes is ω, while the penalty rate is proportional to the level of the evaded tax and equal to $p\tau$, where p is constant and greater than one and the product of ω times p is less than one.

According to our assumptions, unemployed individuals cannot work in the shadow economy and employed individuals cannot be employed in both sectors (formal–

[4] The wage differentials in the informal sector may capture the income uncertainty in this sector.

[5] Since a_1 and a_2 are indepedently distributed, their joint density will be the product of their marginal densities.

informal). It is obvious that there are situations in real life, where these assumptions are not true. However, such assumptions cannot be incorporated into the limited artificial environment of our model. The policy implications we obtain are logical and well-defined even without these assumptions. Nevertheless, the above cases can be considered as a topic for future research.

Workers and vacancies meet each other randomly according to a Pissarides-type matching function, $m(u,v)$, where u is the unemployment rate (since the population of workers is normalized to one) and v is the measure of vacancies. Moreover, we assume that the matching function exhibits constant returns to scale.[6] Hence the arrival rate for workers is $m(\theta)$ where $\theta = v/u$ is the measure of labour market tightness. The usual properties hold for $m(\theta)$, i.e., $m'(\theta) > 0$ and $\lim_{\theta \to 0}[m(\theta)] = 0$. The arrival rate for jobs is $m(\theta)/\theta$ with $[m(\theta)/\theta]' < 0$ and $\lim_{\theta \to 0}[m(\theta)/\theta] = \infty$. Let φ denote the fraction of sector 1 vacancies. Hence, unemployed individuals meet sector 1 vacancies at rate $m(\theta)\varphi$ whereas the contact rate for sector 2 vacancies is $m(\theta)(1-\varphi)$.

2.2 DECISION MAKING

For a worker with ability vector a, let $U(a)$ be the value of unemployment, $W_i(a)$ be the value of employment in a job of sector i, $i = 1,2$, $J_i(a)$ be the value to the employer of filling a job in sector i, $z_i(a)$ be the probability that a match occurs when a worker with ability vector a meets a vacancy in sector i, $\rho_i(a)$ be the probability that the worker is willing to get employed by a sector i job and finally V_i be the value of a vacancy of sector i.

Workers

a) Unemployed

The Bellman equation for an unemployed worker is

$$rU(a) = m(\theta)\varphi z_1(a) max\{W_1(a) - U(a), 0\} + m(\theta)(1-\varphi)z_2(a) max\{W_2(a) - U(a), 0\} \qquad (1)$$

According to the above equation, the flow value of unemployment for a worker endowed with ability vector a, is equal to the arrival rate of sector 1 vacancies willing to employ him times the maximum between the capital gain from being employed on sector 1 job and zero, plus the arrival rate of sector 2 vacancies willing to employ this individual times the maximum between the capital gain from working in sector 2 and zero.

b) Employed

The flow value of employment for a worker with ability a on a job of sector i is

[6] Most empirical studies, such as Anderson and Burgess (2000), Coles and Smith (1996) and Burda (1993) find that a matching function with constant returns to scale fits the data quite well.

$$rW_i(a) = w_i(a) + \delta[U(a) - W_i(a)] \qquad i = 1,2 \tag{2}$$

where $w_i(a)$ is the wage received by a worker with skill vector a, employed to sector i. Equation (2), determines the flow value of employment as the sum of the flow return to employment (the wage) plus the instantaneous capital loss.

Firms

a) Vacant

The Bellman equation for vacancies is

$$rV_i = -c + \frac{m(\theta)}{\theta} E_a[\rho_i(a)\max\{J_i(a) - V_i, 0\}] \tag{3}$$

Equation (3) incorporates the assumption that a is unknown to vacancies before they contact workers and it is only realized when the meeting is taking place. However, firms know the distribution of a s'. Thus they form expectations about their capital gain from becoming filled.

b) Filled

The flow value to a firm in sector i filled by a worker of type a is

$$rJ_i(a) = [a_i - w_i(a)](1 - \omega_i p_i \tau) + \delta[V_i - J_i(a)] \tag{4}$$

where $\omega_1 p_1 = 1$ and $\omega_2 p_2 = \omega p$.

Wage Formation and Reservation Ability

Define $\hat{W}_i(a,w)$ as the value of employment in sector i on wage w. If there is positive surplus, then

$$r\hat{W}_i = w + \delta[U(a) - \hat{W}_i] \Rightarrow \hat{W}_i = \frac{w + \delta U(a)}{r + \delta}$$

Define $\hat{J}_i(a,w)$ as the value of a filled vacancy in sector i on wage w. If there is positive surplus, then

$$r\hat{J}_i = (1 - \omega_i p_i \tau)[a_i - w] + \delta[V_i - \hat{J}_i] \Rightarrow \hat{J}_i = \frac{(1 - \omega_i p_i \tau)[a_i - w] + \delta V_i}{r + \delta}$$

Symmetric Nash Bargaining

$$w = \arg\max_w [\hat{W}_i(a,w) - U(a)][\hat{J}_i(a,w) - V_i]$$

$$\Rightarrow \frac{1}{r+\delta}[\hat{J}_i(a,w) - V_i] = \frac{1 - \omega_i p_i \tau}{r + \delta}[\hat{W}_i(a,w) - U(a)]$$

Then at the equilibrium wage w^*, $J_i = \hat{J}_i$, $W_i = \hat{W}_i$ satisfing

$$W_i(a) - U(a) = \frac{J_i(a) - V_i}{1 - \omega_i p_i \tau} \tag{5}$$

The above condition implies that firms and workers have the same bargaining power. Given the free entry assumption, $V = 0$, equation (5) becomes

$$(1 - \omega_i p_i \tau)[W_i(a) - U(a)] = J_i(a) \tag{6}$$

Equation (6) implies that workers and firms have the same decision rule, i.e., if a worker is willing to get employed by a firm in sector i then $z_i(a) = 1$ otherwise $z_i(a) = 0$ (a match is formed when there is a positive surplus to the match). By using equations (6), (2) and (4), we get that the wage earned by an individual of type a employed in sector i is

$$w_i(a) = \frac{1}{2}[a_i + rU(a)] \tag{7}$$

By using equations (7) and (2) we obtain

$$W_i(a) = \frac{1}{2}[\frac{a_i}{r+\delta}] + [\frac{(r/2)+\delta}{r+\delta}]U(a)$$

A match between an individual with ability vector a with a vacancy of sector i will take place if and only if $a_i \geq rU(a)$ where $i = 1,2$. We get this result by using (7), (2) and the fact that a match will occur if $W_i(a) \geq U(a)$. If $a_1 \geq (<)\ a_2$ and the latter condition holds for a_2 (a_1) then it is implied that it will hold for a_1 (a_2) too.

Lemma 1 *Workers will always accept at least one type of job.*

<u>Proof</u>: If a worker never takes a job, $U(a) = 0$. But $W_i(a) = \frac{1}{2}[\frac{a_i}{r+\delta}] > 0$ implies that a positive matching surplus exists which contradicts the argument that a worker will reject any type of job.[7]

Lemma 2 *Individuals accept only sector 1 jobs if and only if* $a_2 \leq a_2^R(a_1) = \dfrac{m\varphi a_1}{2(r+\delta)+m\varphi}$, *where* $0 \leq a_1 \leq 1$.

[7]As it is easily noted, for a_1, $a_2 = 0$ individuals are indifferent between accepting or rejecting employment. In such a case, we will assume that individuals accept employment.

Proof: Suppose that $a_1 \geq a_2$. This implies that $W_1(a) \geq W_2(a)$. It can be easily shown that there is an $a_2 = a_2^R$, such that

$$W_2(a_1, a_2^R) = U(a_1, a_2^R) \Rightarrow rU(a_1, a_2^R) = a_2^R$$

From equation (1) we get:

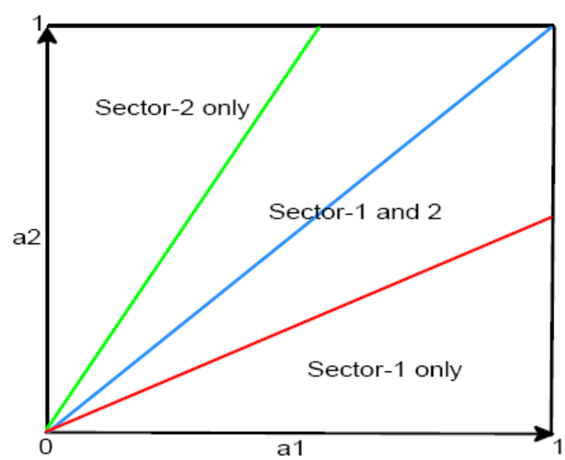
$$rU(a_1, a_2^R) = m(\theta)\varphi[W_1(a_1, a_2^R) - U(a_1, a_2^R)]$$

By using equation (2) and (7), we can show that $a_2^R(a_1) = \dfrac{m\varphi a_1}{2(r+\delta)+m\varphi}$. Hence if $a_1 \geq a_2 \geq a_2^R(a_1)$, then $W_2(a) \geq U(a)$ and individuals accept jobs in both sectors. However if $a_2 < a_2^R \Rightarrow a_2 < rU(a)$, there is no gain from trade with a sector 2 firm and thus workers do not accept jobs in sector 2. Similarly, for $a_1 < a_2$, we get the following Lemma.

Lemma 3 *Individuals accept only sector 2 jobs if and only if* $a_1 \leq a_1^R(a_2) = \dfrac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}$, *where* $0 \leq a_2 \leq 1$.

Proof: It is similar to that of Lemma 2.
The economic intuition behind Lemmas 2 and 3 is the following. When the arrival rate of job offers is high, then individuals are less willing to accept jobs in which they are less productive (the cost of remaining unemployed now by rejecting such job offers is dominated by the benefit of getting employed in a well-paid job in the near future). The same applies when the layoff rate of filled jobs and/or the rate of time preferences (as expressed by the interest rate) are low.

Diagram 1 : Demography



Lemmas 1, 2 and 3 are illustrated in Diagram 1. The blue line is the 45^{o} degree line. On the horizontal axis is the productive capability (a_1) of each individual in sector 1 and on the vertical axis is the corresponding capability (a_2) in sector 2. The green (red) line is the 'frontier' above (below) which individuals accept jobs only in sector 2 (1).

Following from Lemmas 1, 2 and 3 when the arrival rate of job offers increases and the job's destruction rate and/or the interest rate decreases, the red (green) line shifts upwards (downwards). Moreover, when the arrival rate of sector 1 (2) employment opportunities goes up, the red (green) line shifts upwards (downwards).

By using Lemmas 1, 2 and 3 and equations (1), (2) and (7) we get the following flow values of unemployment:

$$rU(a) = \frac{m(.)\phi a_1}{2(r+\delta)+m(.)\phi} \quad \text{if} a_2 \le a_2^R \tag{8}$$

$$rU(a) = \frac{m(.)(1-\phi)a_2}{2(r+\delta)+m(.)(1-\phi)} \quad \text{if} a_1 \le a_1^R \tag{9}$$

$$rU(a) = \frac{m(.)[\phi a_1 + (1-\phi)a_2]}{2(r+\delta)+m(.)} \quad \text{otherwise} \tag{10}$$

The flow value of unemployment for workers with a's below the reservation values depends only on their preferred a (a_1 or a_2). On the other hand, a worker with ability vector that enables him to accept any employment opportunity has his flow value depending on his joint ability vector.

Using equations (4), (7), (8), (9) and (10), and the free entry condition $V_i = 0$, we can easily derive the following values for filled jobs:

$$J_1(a) = \frac{a_1(1-\tau)}{2(r+\delta)+m(.)\varphi} \text{if} a_2 \le a_2^R \tag{11}$$

$$J_2(a) = \frac{a_2(1-\omega p\tau)}{2(r+\delta)+m(.)(1-\varphi)} \text{if} a_1 \le a_1^R \tag{12}$$

$$J_1(a) = \frac{[2(r+\delta)a_1 + m(1-\varphi)(a_1-a_2)](1-\tau)}{2(r+\delta)[2(r+\delta)+m]} \text{if} a_2 \ge a_1 > a_1^R \ \& \ a_1 \ge a_2 > a_2^R \tag{13}$$

$$J_2(a) = \frac{[2(r+\delta)a_2 + m\varphi(a_2-a_1)](1-\omega p\tau)}{2(r+\delta)[2(r+\delta)+m]} \text{if} a_2 \ge a_1 > a_1^R \ \& \ a_1 \ge a_2 > a_2^R \tag{14}$$

3. STEADY STATE

Let $\lambda_t(a)$ and $g_t(a)$ denote the densities of unemployed and employed individuals respectively with skill vector, a, at time t. The above densities are related by the restriction that $f(a) = \lambda_t(a) + g_t(a)$ where $f(a) = 1$ is the density of the total population which does not depend on time. During any infinitely small interval of time, dt, unemployed individuals with $a_2 \leq a_2^R(a_1)$ and $0 \leq a_1 \leq 1$ become employed at rate $m(\theta)\varphi dt$ whereas a fraction δdt of them lose their job. Hence, the evolution of employed individuals with $0 \leq a_2 \leq a_2^R(a_1)$ and $0 \leq a_1 \leq 1$ will be equal to $m(\theta)\varphi\lambda_t(a)dt - \delta g_t(a)dt$. Similarly, we can define the evolution of employed individuals with $0 \leq a_1 \leq a_1^R$, $1 \geq a_2 \geq 0$, and $\tilde{a}_1 \leq a_1 \leq 1$, $a_2^R < a_2 \leq 1$ and $a_2^R(a_1) < a_2 < a_1^{R^{-1}}$, $0 \leq a_1 \leq \tilde{a}_1$ (where $a_1^{R^{-1}}(\tilde{a}_1) = 1$ and $a_1^{R^{-1}}$ is the inverse function of a_1^R). In steady-state the evolution of employed individuals is equal to zero (i.e., the flow of workers out of unemployment should be equal to the flow of workers back to unemployment). Hence, the steady-state distribution of employed individuals is given by (where the use of $f(a) = \lambda_t(a) + g_t(a) = 1$ has been made):

$$g(a) = \frac{m(\theta)\eta(a)}{m(\theta)\eta(a) + \delta} \tag{15}$$

where

$$\eta(a) = \begin{cases} \varphi & \text{for } 0 \leq a_2 \leq a_2^R, 0 \leq a_1 \leq 1 \\ 1 & \text{for } \tilde{a}_1 \leq a_1 \leq 1, a_2^R < a_2 \leq 1 \,\&\, a_2^R < a_2 < a_1^{R^{-1}}, 0 \leq a_1 \leq \tilde{a}_1 \\ 1-\varphi & \text{for } 0 \leq a_1 \leq a_1^R, 1 \geq a_2 \geq 0 \end{cases}$$

Following the same procedure, we get the steady-state distribution of unemployed individuals which is equal to

$$\lambda(.) = \delta/[\delta + \eta(a)m(\theta)] \tag{16}$$

By integrating $\lambda(.)$ and dividing by the steady-state unemployment u we get the proportion of unemployed individuals in each of the above cases. For example the proportion of unemployed individuals who accept any job offer is

$$\left(\int_0^1 \psi(a_1 - a_2^R)da_1 + \int_0^1 \psi(a_2 - a_1^R)da_2\right)/u, \text{where}\, \psi = \delta/(m+\delta).$$

Among all individuals, those who accept employment only in one sector suffer more unemployment. This is reasonable since such workers receive 'worthwhile' offers at a slower rate. The steady state unemployment is given by

$$u = \int_{a_2=0}^{1}\int_{a_1=0}^{1} \lambda(a) da_1 da_2$$

By doing the calculations, we obtain

$$\begin{aligned} u(\theta,\varphi) \quad = \quad & \frac{\delta m}{2}\{\frac{(1-\varphi)}{[2(r+\delta)+m(1-\varphi)][m(1-\varphi)+\delta]} + \frac{\varphi}{[2(r+\delta)+m\varphi](m\varphi+\delta)}\} + \\ & (r+\delta)\psi\{\frac{4(r+\delta)+m}{[2(r+\delta)+m\varphi][2(r+\delta)+m(1-\varphi)]}\} \end{aligned} \tag{17}$$

The steady-state employment can be defined as $1-u$.

Definition 1 *A steady-state equilibrium is a five tuple* a_2^R, a_1^R, θ, u, φ *that satisfy: (i) 'Free' entry, i.e.,* $V_i = 0$, $i = 1,2$, *(ii) 'Balanced flows,' i.e., the flow of workers out of unemployment equals to the flow of workers into unemployment (equation (17)) and (iii) the reservation properties in Lemmas 2 and 3.*

Let $F(a_1, a_2)$ denote the cumulative distribution function (c.d.f.) describing the distribution of a across unemployed workers. Then:

$$\frac{\partial^2 F}{\partial a_1 \partial a_2} = \frac{\lambda(a)}{u}$$

The free entry conditions can be written as:

$$c = \frac{m(\theta)}{\theta}\int_{a_1=0}^{1}\int_{a_2=a_2^R}^{1} J_1(a)\frac{\partial^2 F}{\partial a_1 \partial a_2} da_2 da_1$$

$$c = \frac{m(\theta)}{\theta}\int_{a_2=0}^{1}\int_{a_1=a_1^R}^{1} J_2(a)\frac{\partial^2 F}{\partial a_1 \partial a_2} da_1 da_2$$

From equations (3), (11), (12), (13), (14), (16) and Lemmas 1, 2 and 3 the free entry condition for each sector can be written as (where $m = m(\theta)$):

$$\begin{aligned} c \quad = \quad & \frac{m(1-\tau)}{\theta u(\theta,\varphi)}\{\int_0^1 \frac{a_1 m\varphi a_1 \delta}{[2(r+\delta)+m\varphi]^2(m\varphi+\delta)} da_1 + \int_0^{a_1^R(1)}\int_{a_2^R(\theta,\varphi)}^{a_1^{R^{-1}}(\theta,\varphi)} \mu da_2 da_1 + \\ & \int_{a_1^R(1)}^{1}\int_{a_2^R(\theta,\varphi)}^{1} \mu da_2 da_1\} \end{aligned} \tag{18}$$

where $\mu = \frac{[2(r+\delta)a_1 + m(1-\varphi)(a_1-a_2)]\psi}{2(r+\delta)[2(r+\delta)+m]}$ and $a_1^R(1) = \frac{m(1-\varphi)}{2(r+\delta)+m(1-\varphi)}$.

$$c = \frac{m(1-\omega p\tau)}{\theta u(\theta,\varphi)}\{\int_0^1 \frac{a_2[m(1-\varphi)a_2]\delta}{[2(r+\delta)+m(1-\varphi)]^2[m(1-\varphi)+\delta]}da_2 +$$

$$\int_0^{a_2^R(1)}\int_{a_1^R(\theta,\varphi)}^{a_2^{R^{-1}}(\theta,\varphi)} \zeta da_1 da_2 + \int_{a_2^R(1)}^1\int_{a_1^R(\theta,\varphi)}^1 \zeta da_1 da_2\} \tag{19}$$

where $\zeta = \frac{[2(r+\delta)a_2 + m\varphi(a_2-a_1)]\psi}{2(r+\delta)[2(r+\delta)+m]}$ and $a_2^R(1) = \frac{m\varphi}{2(r+\delta)+m\varphi}$.

Equations (18) and (19) equate the cost of holding a vacancy with the expected revenue from filling it (where the expected revenue is equal to the sum of products of the arrival rate of unemployed with the conditional expectation of a job's net worth). The equilibrium values of θ and φ are given by the solution of the system of (18) and (19) (for the proof of the existence of the equilibrium see the Appendix).

4. COMPARATIVE STATICS

This section analyzes the impact of an increase in profit taxes in sector 1 on the steady-state equilibrium. More specifically, it analyzes the comparative static effects of profit tax on reservation values, unemployment, welfare of individuals and the size of the underground economy. Let Z be the right-hand side of equation (18) and Γ be the right-hand side of equation (19). In order to find the comparative static effects, totally differentiate equations (18) and (19):

$$0 = \frac{\partial Z}{\partial \theta}d\theta + \frac{\partial Z}{\partial \varphi}d\varphi + \frac{\partial Z}{\partial \tau}d\tau$$

$$0 = \frac{\partial \Gamma}{\partial \theta}d\theta + \frac{\partial \Gamma}{\partial \varphi}d\varphi + \frac{\partial \Gamma}{\partial \tau}d\tau$$

By rearranging, and dividing by $d\tau$ we get

$$\frac{\partial Z}{\partial \theta}\frac{d\theta}{d\tau} + \frac{\partial Z}{\partial \varphi}\frac{d\varphi}{d\tau} = -\frac{\partial Z}{\partial \tau} \tag{20}$$

$$\frac{\partial \Gamma}{\partial \theta}\frac{d\theta}{d\tau} + \frac{\partial \Gamma}{\partial \varphi}\frac{d\varphi}{d\tau} = -\frac{\partial \Gamma}{\partial \tau} \tag{21}$$

It can be easily shown that $\partial\Gamma/\partial\tau < 0$ and $\partial Z/\partial\tau < 0$. The sign of the partial derivatives of Z (Γ) with respect to the fraction of sector 1 vacancies (φ) and with respect to the measure of labour tightness (θ) cannot be determined without giving certain values to the parameters of the model. If we evaluate them at $\varphi = 0.5$ (0.5 is the steady-state value of φ, when $\tau = 0$ and the model is symmetric – for τ = 0, equations (18), (19) are equal if $\varphi = 0.5$)[8], and if we assume that the elasticity of $m(.)$ with respect to θ is less than or equal to 0.5 (in the case of a Cobb–Douglas matching function characterized by constant returns to scale, $m(\theta) = A\theta^{\gamma}$ where γ is the elasticity of $m(\theta)$ with respect to θ and A is a parameter that captures the matching efficiency) then $\partial Z/\partial\theta$ ($\partial\Gamma/\partial\theta$) is negative (negative) and $\partial Z/\partial\varphi$ ($\partial\Gamma/\partial\varphi$) is negative (positive) for any value of the parameters (see the Appendix).

According to the preceding analysis, equations (20) and (21) can be written as

$$\begin{bmatrix} \underset{(-)}{\dfrac{\partial Z}{\partial\theta}|_{\varphi=0.5}} & \underset{(-)}{\dfrac{\partial Z}{\partial\varphi}|_{\varphi=0.5}} \\ \underset{(-)}{\dfrac{\partial\Gamma}{\partial\theta}|_{\varphi=0.5}} & \underset{(+)}{\dfrac{\partial\Gamma}{\partial\varphi}|_{\varphi=0.5}} \end{bmatrix} \begin{bmatrix} \dfrac{d\theta}{d\tau}|_{\varphi=0.5} \\ \dfrac{d\varphi}{d\tau}|_{\varphi=0.5} \end{bmatrix} = \begin{bmatrix} \underset{(+)}{-\dfrac{\partial Z}{\partial\tau}|_{\varphi=0.5}} \\ \underset{(+)}{-\dfrac{\partial\Gamma}{\partial\tau}|_{\varphi=0.5}} \end{bmatrix} \tag{22}$$

where the brackets below the partial derivatives indicate their sign. By solving (22), we get that $\dfrac{d\theta}{d\tau}|_{\varphi=0.5}, \dfrac{d\varphi}{d\tau}|_{\varphi=0.5} < 0$ (For φ = 0.5, it can be easily derived that $\dfrac{\partial\Gamma}{\partial\theta}|_{\varphi=0.5}\dfrac{\partial Z}{\partial\tau}|_{\varphi=0.5} - \dfrac{\partial Z}{\partial\theta}|_{\varphi=0.5}\dfrac{\partial\Gamma}{\partial\tau}|_{\varphi=0.5} > 0$). These total derivatives imply that if we start from the symmetric case where $\tau = 0$ and increase τ, then the new steady-state equilibrium will be characterized by lower θ and φ.

As τ increases, the right hand side of equations (18) and (19) decreases while the left hand side (cost of holding a vacancy) remains unchanged. However, the right-hand side of equation (18) decreases at a faster rate. As a result of this change, sector 1 and 2 will reduce their vacancy supply until the zero profit condition (equations (18), (19)) is restored. This reaction will have two effects: (i) the reduction of the vacancy-unemployment ratio ($\theta = v/u$) since the total number of vacancies decreases and (ii) the change of the mix of vacancies in favour of the underground sector (decrease of φ). This result is driven by the fact that equation (18) decreases at a faster rate.

As was previously shown, the reduction of φ has a positive impact on the right hand side of equation (18)[9]. The positive effect from the decrease of the fraction of sector 1 vacancies will mitigate the negative one from the increase of profit taxation, and therefore will confine the reduction of sector 1 vacancies and consequently of the measure of labour market tightness. However, the negative effect will prevail and the vacancy-unemployment ratio will be reduced. The reaction of sector 1 in profit taxation will have an immediate impact on the underground sector. The decrease of θ

[8] By evaluating the partial derivative of Z (Γ) w.r.t. φ at $\phi=0.5$, we get a good approximation for the change that occurs on Z (Γ) as φ deviates from 0.5.

[9] As φ decreases the number of unemployed searching only for sector 1 jobs increases (since $\delta/(\delta + m(\theta)\varphi)$ decreases) and their reservation ability and consequently their threat point in the bargaining process decreases. These two effects lead to the increase of the expected revenue of sector 1 firms.

as a result of the lower vacancy supply in sector 1 will give an incentive in sector 2 to increase its vacancy creation. However, this effect will be completely offset by the negative effect caused by the change in the mix of vacancies in favour of the shadow sector[10] and the increase in tax rates. At the end the steady-state measure of labour market tightness and the fraction of sector 1 vacancies will diminish. A number of individuals previously searching only for sector 1 jobs will now accept offers in both sectors as a_2^R unambiguously decreases.[11] The impact that the taxation of profits has on a_1^R is ambiguous. The reason for that is the existence of two opposing effects; the 'tightness' effect (reduction of θ) which decreases a_1^R and the 'composition' effect (reduction of φ) which increases a_1^R. More people will accept jobs only in the underground sector if and only if

$$\frac{\frac{d\theta}{d\tau}|_{\varphi=0.5}}{\frac{d\varphi}{d\tau}|_{\varphi=0.5}} < \frac{m(\theta)}{m^{'}(\theta)(1/2)} = \frac{2\theta}{\kappa(\theta)} \tag{23}$$

where $\kappa(\theta)$ is the elasticity of $m(\theta)$ with respect to θ.

If inequality (23) holds, then the percentage of individuals working in the underground sector will be greater after the increase in the profit tax. Moreover, if (23) holds then the arrival rate of sector 2 ($m(\theta)(1-\varphi)$) jobs increases as we raise the profit tax ($\frac{d[m(\theta)(1-\varphi)]}{d\tau} = m^{'}(\theta)\varphi\frac{d\theta}{d\tau} - m(\theta)\frac{d\varphi}{d\tau}$). By assuming a Cobb–Douglas matching function characterized by constant returns to scale and simulating the model, we can show that (23) does not hold if the parameter which captures the matching technology (constant of matching) is quite high (the constant of matching can be increased through active labour market policies). This occurs since the higher the value of the parameter which captures the 'technological' advances in the matching process, the lower is the positive effect caused by the increase of φ-the high value of the constant of matching, will mitigate the negative impact of taxation on the fraction of sector 1 vacancies (for high values of this parameter, the positive effect from φ is dominated by the negative effect from θ). However, even if (23) does not hold, then if we start from the symmetric case ($\varphi = 0.5,\ \ \tau = 0$) and raise the tax the percentage of individuals working in the underground sector will again be greater. This occurs due to the fact that the decrease of a_2^R is greater than the decrease of a_1^R and because an individual who accepts jobs in both sectors, is more likely to be offered a sector 2 job (as a result of the decrease in φ). Table 1, presents the results of a simulation of the model described above when (23) holds. In Table 1, the baseline values of the

[10] By bringing the cost of holding a vacancy, c on the right-hand side of equation (18) and total differentiating, we get that $\frac{\partial\Gamma}{\partial\theta}|_{\phi=0.5}d\theta = -\frac{\partial\Gamma}{\partial\phi}|_{\phi=0.5}d\phi$, where the partial derivatives are evaluated in the case where the profit tax is equal to zero (symmetric case).

[11] a_2^R is increasing in φ and θ.

parameters are: $A = 0.5$, $c = 0.3$, $\delta = 0.1$, $r = 0.05$, $\omega = 0.3$, $p = 1.5$ and $m(\theta) = A\sqrt{\theta}$ (where A is the parameter capturing the technological advances in the matching process). Finally, Table 2 presents a case where (23) does not hold ($A = 1.5$, $c = 0.3$, $\delta = 0.1$, $r = 0.05$, $\omega = 0.3$, $p = 1.5$ and $m(\theta) = A\theta^{0.5}$). Our parameter values where chosen so as to produce plausible results for our baseline case where there are no taxes.

Table 1: *Simulation of the model when (23) holds*

τ	θ	φ	u	a_2^R	a_1^R	% s.1 empl.	% s.2 empl.
0	1.115	0.5	0.213	0-0.4681	0-0.4681	0.5 (0.3935)	0.5 (0.3935)
0.05	1.063	0.487	0.217	0-0.4556	0-0.4685	0.488 (0.382)	0.512 (0.401)
0.1	1.012	0.471	0.22	0-0.4412	0-0.47	0.473 (0.369)	0.527 (0.411)
0.15	0.961	0.454	0.224	0-0.4259	0-0.4715	0.457 (0.355)	0.543 (0.421)
0.2	0.91	0.434	0.227	0-0.4083	0-0.4736	0.438 (0.339)	0.562 (0.434)

The last two columns of Tables 1 and 2 present the fraction of sector 1 and sector 2 employed (where inside the brackets is the absolute number of employed). The fourth and the fifth column present the range of the reservation abilities.

Table 2: *Simulation of the model when (23) does not hold*

τ	θ	φ	u	a_2^R	a_1^R	% s.1 empl.	% s.2 empl.
0	1.718	0.5	0.082	0-0.7662	0-0.7662	0.5 (0.459)	0.5 (0.459)
0.05	1.648	0.495	0.083	0-0.761	0-0.7642	0.498 (0.457)	0.502 (0.46)
0.1	1.578	0.489	0.085	0-0.754	0-0.7624	0.494 (0.452)	0.506 (0.463)
0.15	1.508	0.482	0.087	0-0.747	0-0.761	0.491 (0.45)	0.509 (0.465)
0.2	1.437	0.475	0.088	0-0.74	0-0.759	0.487 (0.44)	0.513 (0.468)

As we observe unemployment increases in τ. The main reason for that is the decrease in θ. When labour market tightness decreases, unemployment increases due to the decrease of the contact rate.

However, there is a positive effect stemming from the fact that individuals become less picky. If

$$min[2a_1\delta\varphi(r+\delta) - a_1m^2(.)\varphi^2, 2a_1\delta(1-\varphi)(r+\delta) - a_1m^2(.)(1-\varphi)^2] \leq 0$$

(i.e., as frictions decline or as we approach the classical model) then the positive effect is dominated by the negative effect and unemployment increases.

As was discussed above, when the constant of matching (A) and consequently the contact rate among economic agents is high enough, then the reservation productivity of those accepting only informal sector jobs decreases. This result is the opposite with the case of firing and payroll tax in Albrecht et al (2006). This result occurs since in our formulation the arrival rate of informal sector jobs is endogenous. Hence, our analysis implies that a corporate income tax can lead to the opposite reservation results[12] regarding informal sector as a firing or a payroll tax increases, as long as matching technology is relatively high (high matching technology can be obtained through active labour market policies). This effect will mitigate the expansion of the underground sector. In other words, active labour market policies assisting the matching process will limit the negative effects of taxation. Moreover, the average wage decreases with corporate income tax regardless of inequality (23) but when (23) does not hold –the constant of matching is high– then the decrease is smoother. This result is illustrated in Diagrams 2 and 3.

Diagram 2

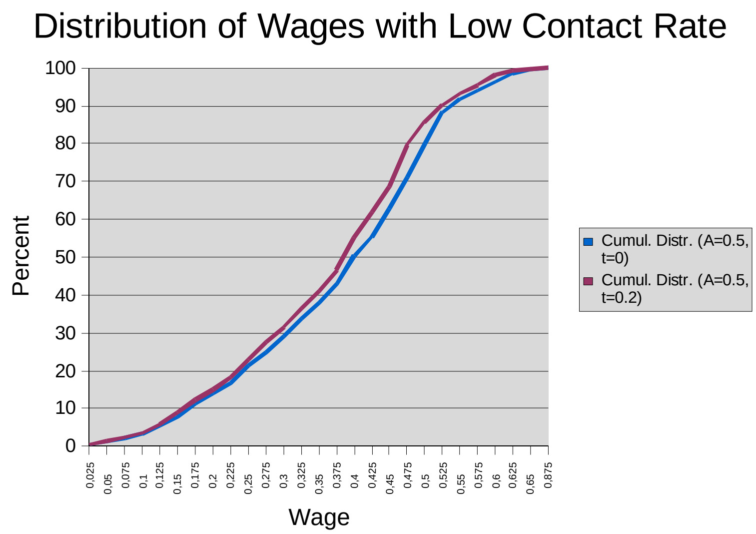
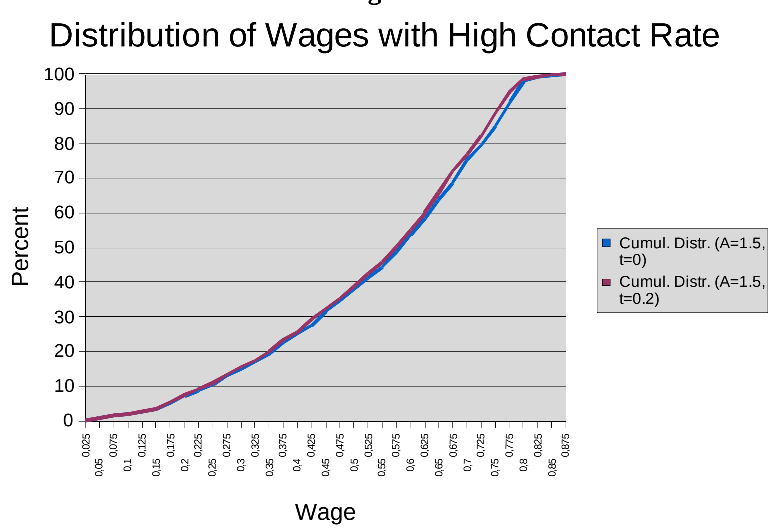


Diagram 3



[12] The number of people searching only for informal sector jobs will decrease.

Our welfare analysis will focus on the individuals who accept jobs only in the underground sector. This is because under certain conditions their welfare increases with profit tax. More specifically, according to our previous analysis, if (23) holds then a_1^R and the arrival rate of sector 2 vacancies both increase with profit taxation. An immediate result from the increase of a_1^R is the increase of the wage received by the individuals who are employed in sector 2 and their $a_2 \leq a_1^R$ (for those individuals a_1^R represents their reservation wage). Moreover, an increase in the arrival rate of jobs in the underground sector decreases the period of unemployment for those searching for sector 2 jobs. Hence, the welfare of individuals with $a_2 \leq a_1^R$ (accept jobs only in sector 2) after the increase of profit tax unambiguously increases.

4.1 FIRING TAXES

Under a firing tax and without corporate taxes equation (4) becomes

$$rJ_i(a) = [a_i - w_i(a)] + \delta[V_i - J_i(a) - s_i] \tag{4b}$$

where $s_2 = \omega ps$, $s_1 = s$ and s is the firing tax. Mathematical calculations yield

$$J_1(a|\ a_2 \leq a_2^R) = \frac{a_1}{2(r+\delta)+m(.)\varphi} - \frac{\delta s}{r+\delta} \tag{11b}$$

$$J_2(a|\ a_1 \leq a_1^R) = \frac{a_2}{2(r+\delta)+m(.)(1-\varphi)} - \frac{\delta \omega ps}{r+\delta} \tag{12b}$$

$$J_1(a|\ a_2 \geq a_1 > a_1^R \ \&\ a_1 \geq a_2 > a_2^R) = \frac{[2(r+\delta)a_1 + m(1-\varphi)(a_1 - a_2)]}{2(r+\delta)[2(r+\delta)+m]} - \frac{\delta s}{r+\delta} \tag{13b}$$

$$J_2(a|\ a_2 \geq a_1 > a_1^R \ \&\ a_1 \geq a_2 > a_2^R) = \frac{2(r+\delta)a_2 + m\varphi(a_2 - a_1)}{2(r+\delta)[2(r+\delta)+m]} - \frac{\delta \omega ps}{r+\delta} \tag{14b}$$

The flow values of unemployment are the same with these in the analysis of corporate taxation. Hence, under a firing tax the equilibrium values of θ and φ are given from the following equations:

$$\begin{aligned} c \quad = \quad & \frac{m}{\theta u(\theta,\varphi)}\{[\int_0^1 \frac{a_1 m\varphi a_1 \delta}{[2(r+\delta)+m\varphi]^2(m\varphi+\delta)} da_1 + \int_0^{a_1^R(1)}\int_{a_2^R(\theta,\varphi)}^{a_1^{R^{-1}}(\theta,\varphi)} \mu\, da_2 da_1 + \\ & \int_{a_1^R(1)}^1 \int_{a_2^R(\theta,\varphi)}^1 \mu\, da_2 da_1]\} \\ & - \frac{\delta s}{(r+\delta)}\{[\int_0^1\int_0^{a_1^R(\theta,\varphi)} \frac{\delta}{m\varphi+\delta} da_2 da_1 + \int_0^{a_1^R(1)}\int_{a_2^R(\theta,\varphi)}^{a_1^{R^{-1}}(\theta,\varphi)} \frac{\delta}{m+\delta} da_2 da_1 \\ & + \int_{a_1^R(1)}^1\int_{a_2^R(\theta,\varphi)}^1 \frac{\delta}{m+\delta} da_2 da_1]\} \end{aligned} \tag{18b}$$

$$\begin{aligned} c \quad = \quad & \frac{m}{\theta u(\theta,\varphi)}\{\int_0^1 \frac{a_2[m(1-\varphi)a_2]\delta}{[2(r+\delta)+m(1-\varphi)]^2[m(1-\varphi)+\delta]}da_2 + \int_0^{a_2^R(1)}\int_{a_1^R(\theta,\varphi)}^{a_2^{R^{-1}}(\theta,\varphi)} \zeta da_1 da_2 + \\ & \int_{a_2^R(1)}^1\int_{a_1^R(\theta,\varphi)}^1 \zeta da_1 da_2\} - \frac{\delta\omega ps}{(r+\delta)}\{[\int_0^1\int_0^{a_2^R(\theta,\varphi)} \frac{\delta}{m(1-\varphi)+\delta}da_1 da_2 + \int_0^{a_2^R(1)}\int_{a_1^R(\theta,\varphi)}^{a_2^{R^{-1}}(\theta,\varphi)} \frac{\delta}{m+\delta}da_1 da_2 \\ & + \int_{a_2^R(1)}^1\int_{a_1^R(\theta,\varphi)}^1 \frac{\delta}{m+\delta}da_1 da_2]\} \end{aligned} \tag{19b}$$

The existence of equilibrium can be easily proven (see the Appendix). Following the same procedure with the above subsection, we get $\frac{d\theta}{ds}|_{\varphi=0.5}, \frac{d\varphi}{ds}|_{\varphi=0.5} < 0$ (see the Appendix). Hence, an increase in firing tax (when corporate tax is zero) has the same effects as the increase in corporate taxation. In the case of severance taxes, equation (23) becomes

$$\frac{\frac{d\theta}{ds}|_{\varphi=0.5}}{\frac{d\varphi}{ds}|_{\varphi=0.5}} < \frac{m(\theta)}{m'(\theta)(1/2)} = \frac{2\theta}{\kappa(\theta)} \tag{23b}$$

In Albrecht et al (2006) an increase in firing tax will reduce the level of the unemployment rate. In our model the exact opposite result occurs (i.e. unemployment increases in firing tax). This result occurs, because in our analysis the reduction in the job arrival rate is not outweighed by the increasing job duration since it is assumed that there is no endogenous job destruction.

Table 3 presents the results of a simulation of the model described above when (23b) holds. In Table 3, the baseline values of the parameters are $A = 0.5$, $c = 0.3$, $\delta = 0.1$, $r = 0.05$, $\omega = 0.3$, $p = 1.5$ and $m(\theta) = A\sqrt{\theta}$. Table 4 presents the case where (23b) does not hold ($A = 1.5$, $c = 0.3$, $\delta = 0.1$, $r = 0.05$, $\omega = 0.3$, $p = 1.5$ and $m(\theta) = A\theta^{0.5}$).

Table 3: *Simulation of the model when (23b) holds*

s	θ	φ	u	a_2^R	a_1^R	% s.1 empl.	% s.2 empl.
0	1.115	0.5	0.213	0-0.4681	0-0.4681	0.5 (0.3935)	0.5 (0.3935)
0.01	1.107	0.498	0.214	0-0.4662	0-0.4682	0.498 (0.3914)	0.502 (0.3946)
0.03	1.092	0.494	0.215	0-0.4625	0-0.4684	0.494 (0.3878)	0.505 (0.3964)
0.08	1.055	0.485	0.217	0-0.454	0-0.4685	0.486 (0.3805)	0.514 (0.4025)

Table 4: *Simulation of the model when (23b) does not hold*

s	θ	φ	u	a_2^R	a_1^R	% s.1 empl.	% s.2 empl.
0	1.718	0.5	0.082	0-0.7662	0-0.7662	0.5 (0.459)	0.5 (0.459)
0.01	1.698	0.499	0.082	0-0.7648	0-0.7655	0.4995 (0.4585)	0.5005 (0.4594)
0.03	1.658	0.495	0.083	0-0.7612	0-0.7648	0.4975 (0.4562)	0.5025 (0.4608)
0.08	1.562	0.488	0.085	0-0.753	0-0.7619	0.494 (0.452)	0.506 (0.463)

The last two columns of Tables 3 and 4 present the fraction of sector 1 and sector 2 employed (where inside the brackets is the absolute number of employed). The fourth and the fifth column present the range of the reservation abilities.

5. CONCLUSION

In this paper, we examined how profit and firing taxation influence the size of the underground economy. We conclude that the impact of wage and payroll taxation on the size of the underground sector (a subject which is widely examined by the literature) is the same with that of profit and firing taxation. More specifically as profit tax or severance tax increases, the size of the underground sector increases too. Moreover, we showed that the adoption of active labour market policies which assist unemployed individuals to find more easily the 'whereabouts' of vacant jobs, will 'mitigate' the expansion of underground sector and the reduction of wages caused by taxation. Finally, active labour market policies can increase the welfare of a subgroup of individuals as taxation (corporate or firing) increases.

REFERENCES

Albrecht, J. and S. Vroman 2002, 'A matching model with endogenous skill requirements,' *International Economic Review* 43(1): 283–305.

Albrecht, J., L. Navarro, and S. Vroman 2006, 'The effects of labor market policies in an economy with an informal sector', *IZA Discussion Paper* No. 2141, Institute for the Study of Labor, Bonn.

Allingham, M. and A. Sandmo (1972), 'Income tax evasion: A theoretical analysis,' *Journal of Public Economics* 1(3–4): 323–338.

Anderson, P. and S. Burgess (2000), 'Empirical matching functions: Estimation and interpretation using state-level data,' *Review of Economics and Statistics* 82(1): 93–102.

Blakemore, A., P. Burgess, S. Low and R. Louis (1996), 'Employer tax evasion in the unemployment insurance program,' *Journal of Labor Economics* 14(2): 210–230.

Burda, M. (1993), 'Unemployment labour markets and structural change in Eastern Europe,' *Economic Policy* 16(April): 101–137.

Clotfelter, C. (1983), 'Tax evasion and tax rates: An analysis of individual returns,' *Review of Economics and Statistics* 65(3): 363–373.

Coles, M., and E. Smith (1996), 'Cross-section estimation of the matching function: Evidence from England and Wales,', *Economica* 63(252): 589–597.

Giles, D. (1999a), 'Measuring the hidden economy: Implications for econometric modelling,' *Economic Journal* 109(456): 370–380.

Kesselman, J. (1989), 'Income tax evasion: An intersectoral analysis,' *Journal of Public Economics* 38(2): 137–182.

Loayza, N. (1996), 'The economics of the informal sector: A simple model and some empirical evidence from Latin America,' *Carnegie-Rochester Conf. Series Public Policy* 45: 129–162.

Mirrlees, J. (1971), 'Notes on some special cases of the taxation of risk-taking,' mimeographed.

Mortensen, D., and C. Pissarides (1994), 'Job creation and job destruction in the theory of unemployment,' *Review of Economic Studies* 61(3): 397–415.

Pissarides, C. (2000), *Equilibrium Unemployment Theory*, 2nd edition, MIT Press, Cambridge, MA.

Roy, A. (1951), 'Some thoughts on the distribution of earnings,' *Oxford Economic Papers* 3(2): 135–146.

Schneider, F., and D. Enste (2000), 'Shadow economies: Size, causes, and consequences,' *Journal of Economic Literature* 38(1): 77–114.

Thomas, J. (1992), *Informal Economic Activity*, LSE Handbooks in Economics, Harvester Wheatsheaf, London.

Yitzhaki, S. (1974), 'A note on income tax evasion: A theoretical analysis,' *Journal of Public Economics* 3(2): 201–202.

APPENDIX

Given that for $\tau = 0$ the model becomes symmetric and $\varphi = 0.5$, the solution is given by

$$c = \frac{m}{\theta u(\theta,0.5)}\{\int_0^1 \frac{a_2[m0.5a_2]\delta}{[2(r+\delta)+m0.5]^2[m0.5+\delta]}da_2 + \int_0^{a_2^R(1)}\int_{a_1^R(\theta,0.5)}^{a_2^{R^{-1}}(\theta,0.5)} \zeta da_1 da_2 + \int_{a_2^R(1)}^1\int_{a_1^R(\theta,0.5)}^1 \zeta da_1 da_2\} \tag{24}$$

where $$u(\theta,0.5) = \frac{2\delta[2\delta(r+\delta)+m(r+\delta)+m0.5(m+\delta)]}{(m+\delta)(m+2\delta)[m0.5+2(r+\delta)]}.$$

As $\theta \to 0$, $\lim_{\theta\to 0}\left(\frac{2\delta[2\delta(r+\delta)+m(r+\delta)+m0.5(m+\delta)]}{(m+\delta)(m+2\delta)[m0.5+2(r+\delta)]}\right)=1$,

$\lim_{\theta\to 0}\frac{[2(r+\delta)a_2+m0.5(a_2-a_1)]\psi}{2(r+\delta)[2(r+\delta)+m]}=\frac{a_2}{2(r+\delta)}$,

$\lim_{\theta\to 0}\frac{a_2[m0.5a_2]\delta}{[2(r+\delta)+m0.5]^2[m0.5+\delta]}=0$,

$\lim_{\theta\to 0}a_2^R(1)=0$,

$\lim_{\theta\to 0}a_1^R(\theta,\varphi)=0$,

$\lim_{\theta\to 0}a_2^R(\theta,0.5)=0$,

$\lim_{\theta\to 0}a_1^R(\theta,0.5)=0$ (since the reservation values are equal to zero as $\theta \to 0$, the first two double integrals of the above equation are equal to zero). From our assumptions $\lim_{\theta\to 0}\frac{m}{\theta}=\infty$. Hence as $\theta\to 0$ the r.h.s of (24) approaches infinity.

As $\theta\to\infty$, the reservation productivities are equal to the 45^{o} line. Hence the last two integrals of (24) are equal to zero. We get that $\lim_{\theta\to\infty}\frac{m}{\theta u(\theta,0.5)}\int_0^1\frac{a_2[m0.5a_2]\delta}{[2(r+\delta)+m0.5]^2[m0.5+\delta]}da_2=$

$$\lim_{\theta\to\infty}\frac{m}{\theta}\lim_{\theta\to\infty}\frac{2[m0.5]}{3[2(r+\delta)+m0.5]}\frac{(m+\delta)}{2[2\delta(r+\delta)+m(r+\delta)+m0.5(m+\delta)]}\overset{\text{by applying del Hospital rule}}{=}0$$

Hence as $\theta\to\infty$ the r.h.s. of (24) approaches zero. Moreover as we have shown the r.h.s. of (24) decreases in θ. The above analysis implies that a unique equilibrium exists for $\tau=0$. The same result is derived in the case of severance tax since $\frac{\delta s}{(r+\delta)}$ does not depend on θ.

Equation (18) describes a downward sloping curve in θ, φ locus ($\frac{\partial Z}{\partial\theta}|_{\varphi=0.5}<0$ and $\frac{\partial Z}{\partial\varphi}|_{\varphi=0.5}<0$ (for the proof see below). On the other hand (19) describes an upward sloping curve in θ, φ locus ($\frac{\partial\Gamma}{\partial\theta}|_{\varphi=0.5}<0$ and $\frac{\partial\Gamma}{\partial\varphi}|_{\varphi=0.5}>0$ (the proof is given below).

By substituting $\varphi=0$ into (18) we obtain

$$c = \frac{\delta(1-\tau)\{6(r+\delta)[m+2(r+\delta)]+m^2\}}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,0)} \Rightarrow$$

$$c = \frac{\delta(1-\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,0)} + \frac{\delta(1-\tau)m^2}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,0)} = \Lambda(\theta)$$

By substituting $\varphi = 0$ into (19) we get:

$$c = \frac{\delta(1-\omega p\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,0)} = \Omega(\theta)$$

The above equations have a solution in θ, since they are decreasing in θ and $\lim_{\theta\to 0}\Lambda(\theta),\Omega(\theta) = \infty$ and $\lim_{\theta\to\infty}\Lambda(\theta),\Omega(\theta) = 0$.

Let θ_1 be the solution of $c = \Omega(\theta)$ and θ_2 be the solution of $c = \Lambda(\theta)$. In order to prove that the curve described by (18) is above that described by (19) for $\varphi = 0, \tau \neq 0$ (i.e. $\theta_2 > \theta_1$), we have to show that:

$$\Lambda(\theta_1) > \Omega(\theta_1)$$

By solving with respect to τ, we get:

$$\tau < \frac{m^2(\theta_1)}{m^2(\theta_1) + 6(r+\delta)[m(\theta_1)+2(r+\delta)](1-\omega p\tau)}$$

The r.h.s. of the above inequality is positive and less than one. Hence, there will be values of τ, such that $\theta_2 > \theta_1$ for $\varphi = 0$. By substituting $\varphi = 1$ into (18) we get:

$$c = \frac{\delta(1-\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)}$$

By substituting $\varphi = 1$ into (19) we get:

$$c = \frac{\delta(1-\omega p\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)} + \frac{\delta(1-\omega p\tau)m^2}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,1)}$$

By following the same analysis, it can be easily shown that the value of θ which satisfies $c = \frac{\delta(1-\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)}$, is less than that satisfying $c = \frac{\delta(1-\omega p\tau)}{2(m+\delta)[m+2(r+\delta)]}\frac{m}{\theta u(\theta,1)} + \frac{\delta(1-\omega p\tau)m^2}{12[m+2(r+\delta)]^2(r+\delta)(m+\delta)}\frac{m}{\theta u(\theta,1)}$. Hence, the existence of solution for $\tau \neq 0$ is proved.

Starting from $\tau = 0$, $\varphi = 0.5$ an increase in τ corresponds to a shift of the curve described by (19) downwards. Hence, if we start from the symmetric case where $\tau = 0$ and $\varphi = 0.5$, and increase τ, there will exist an equilibrium characterized by lower φ and θ. The same analysis is applied in the case of firing taxes.

A) *Proof that* $\partial Z/\partial\theta < 0$ ($\partial\Gamma/\partial\theta < 0$) *when the derivative is calculated for* $\varphi = 0.5$ *and the elasticity of* $m(\theta)$ *w.r.t.* θ *is less or equal to* 0.5.

Z consists of two parts: the arrival rate of workers ($m(\theta)/\theta$) divided by the measure of steady-state unemployment and the term inside the braces. The term inside the braces can be alternatively written with the following way:

$$(1-\tau)\{\int_0^1\int_0^{a_2^R}\frac{a_1\delta}{[2(r+\delta)+m\varphi](m\varphi+\delta)}da_2da_1+\int_0^1\int_{a_2^R}^{a_1}\mu da_2da_1+ \\ \int_0^{a_1^R(1)}\int_{a_1}^{a_1^{-1R}}\mu da_2da_1+\int_{a_1^R(1)}^1\int_{a_1}^1\mu da_2da_1\} \tag{25}$$

where

$$\mu=\frac{[2(r+\delta)a_1+m(1-\varphi)(a_1-a_2)]\psi}{2(r+\delta)[2(r+\delta)+m]}$$

$$\psi=\delta/[m+\delta]$$

$$a_1^R(a_2)=\frac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}$$

$$a_2^R(a_1)=\frac{m\varphi a_1}{2(r+\delta)+m\varphi}$$

$$m=m(\theta)$$

and $a_1^{R^{-1}}$ is the inverse function of a_1^R. In equation (25), the sum of the last two double integrals describe the area between the 45$^{\text{o}}$ line and the $a_1^{R^{-1}}$ (look at Diagram 1). Since the value of a filled job ($J_i(.)$) is always decreasing in θ and a_1^R ($a_1^{R^{-1}}$) is increasing (decreasing) in θ, the derivative of the sum of the last two double integrals with respect to θ will be always negative. By mathematical manipulations, we can show that the sum of the first two double integrals is equal to

$$\int_0^1\frac{a_1^2\delta\varphi m}{[2(r+\delta)+m\varphi]^2[\delta+m\varphi]}da_1+\int_0^1\frac{(r+\delta)a_1^2\delta}{[2(r+\delta)+m\varphi]^2[\delta+m]}da_1+ \\ \int_0^1\frac{(r+\delta)a_1^2\delta}{[2(r+\delta)+m\varphi][\delta+m][2(r+\delta)+m]}da_1 \tag{26}$$

In the above equation, the derivative of the third integral with respect to θ is always negative, whereas the derivative of the sum of the first two integrals with respect to labour market tightness is equal to

$$\frac{\delta m'}{3[2(r+\delta)+m\varphi]^3}\{\frac{\varphi[2(r+\delta)\delta-m\varphi\delta-2m^2\varphi^2]}{[\delta+m\varphi]^2}-\frac{\varphi m(r+\delta)}{[\delta+m]^2}-\frac{\varphi 2(r+\delta)}{[\delta+m]}-\frac{2(r+\delta)^2}{[\delta+m]^2}\} \quad (27)$$

But for $\varphi = 0.5$, $\delta/[\delta+m\varphi]^2$ is less than $1/(m+\delta)$. Hence, the derivative of the term inside the braces in equation (18) w.r.t. θ is always negative if it is evaluated at $\varphi = 0.5$.

The steady-state unemployment is equal to

$$u(\varphi,\theta) \quad = \quad \int_0^1\int_0^{\frac{m\varphi a_1}{2(r+\delta)+m\varphi}}\frac{\delta}{m\varphi+\delta}da_2da_1+\int_0^1\int_{\frac{m\varphi a_1}{2(r+\delta)+m\varphi}}^{a_1}\frac{\delta}{m+\delta}da_2da_1+$$
$$\int_0^1\int_0^{\frac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}}\frac{\delta}{m(1-\varphi)+\delta}da_1da_2+\int_0^1\int_{\frac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}}^{a_2}\frac{\delta}{m+\delta}da_1da_2$$

After simplifying the above expression and by multiplying with θ/m, we can show that $u(\varphi,\theta)\theta/m$ is equal to

$$\int_0^1\frac{\delta\theta a_2}{(m+\delta)m}da_2+\int_0^1\frac{\delta\theta a_1}{(m+\delta)m}da_1+\int_0^1\int_0^{\frac{m(1-\varphi)a_2}{2(r+\delta)+m(1-\varphi)}}\frac{\varphi\theta}{[m(1-\varphi)+\delta](m+\delta)}da_1da_2+$$
$$\int_0^1\int_0^{\frac{m\varphi a_1}{2(r+\delta)+m\varphi}}\frac{(1-\varphi)\theta}{(m\varphi+\delta)(m+\delta)}da_2da_1 \quad (28)$$

By differentiating equation (28) w.r.t. θ, we get that $\partial(u\theta/m)/\partial\theta$ is equal to

$$\frac{\delta[\delta(m-\theta m')+m(m-2\theta m')]}{(m+\delta)^2m^2}+\int_0^1\varphi\{\frac{\theta a_1^{R'}}{[m(1-\varphi)+\delta](m+\delta)}+$$
$$\frac{a_1^R[\delta^2+\delta(2-\varphi)(m-\theta m')+(1-\varphi)(m-2\theta m')]}{[m(1-\varphi)+\delta]^2(m+\delta)^2}\}da_2+$$
$$\int_0^1(1-\varphi)\{\frac{\theta a_2^{R'}}{[m\varphi+\delta](m+\delta)}+\frac{a_2^R[\delta^2+\delta(1+\varphi)(m-\theta m')+\varphi(m-2\theta m')]}{[m\varphi+\delta]^2(m+\delta)^2}\}da_1$$

If we assume that the elasticity of $m(\theta)$ w.r.t. θ is less or equal to 0.5 (i.e. $m-2\theta m' \geq 0$), and given the fact that $m-\theta m' > 0$ (from our initial assumption regarding the properties of the matching function, we get that $\partial(m/\theta)/\partial\theta = (\theta m'-m)/\theta^2 < 0$), $\partial(u\theta/m)/\partial\theta > 0$. Hence, it is proved that

$\partial Z/\partial\theta < 0$ when the derivative is calculated for $\varphi = 0.5$ and the elasticity of $m(\theta)$ w.r.t. θ is less or equal to 0.5 (the proof for $\partial\Gamma/\partial\theta$ is similar).

Since $\frac{\delta s}{(r+\delta)}$ does not depend on θ, φ the analysis and the results in the case of firing taxes is the same.

B) Proof that $\partial Z/\partial\varphi < 0$ ($\partial\Gamma/\partial\varphi > 0$) when the derivative is calculated for $\varphi = 0.5$

Z is the product of the the arrival rate of workers ($m(\theta)/\theta$) divided by the measure of steady-state unemployment times the term inside the braces. The derivative of the steady-state unemployment w.r.t. φ is equal to

$$\frac{\delta m}{2}\{\frac{-2\delta(r+\delta)+m^2(1-\varphi)^2}{[2(r+\delta)+m(1-\varphi)]^2[\delta+m(1-\varphi)]^2}+\frac{2\delta(r+\delta)-m^2\varphi^2}{[\delta+m\varphi]^2[2(r+\delta)+m\varphi]^2}\}+ \\ (r+\delta)\psi\{\frac{-[4(r+\delta)+m]m^2(1-2\varphi)}{[2(r+\delta)+m(1-\varphi)]^2[2(r+\delta)+m\varphi]^2}\} \tag{29}$$

where $\psi = \delta/(m+\delta)$. It can be easily shown that the above equation is equal to zero for $\varphi = 0.5$.

In equation (25), the derivative of the sum of the last two double integrals with respect to φ is equal to

$$\frac{\delta m(r+\delta)}{3[2(r+\delta)+m(1-\varphi)]^3[2(r+\delta)+m](m+\delta)} \tag{30}$$

This equation is always positive.

By mathematical calculations, we can demonstrate that the sum of the first two double integrals in equation (25) is equal to

$$\int_0^1\frac{a_1^2\delta\varphi m}{[2(r+\delta)+m\varphi]^2[\delta+m\varphi]}da_1+\int_0^1\frac{(r+\delta)a_1^2\delta}{[2(r+\delta)+m\varphi]^2[\delta+m]}da_1+ \\ \int_0^1\frac{(r+\delta)a_1^2\delta}{[2(r+\delta)+m\varphi][\delta+m][2(r+\delta)+m]}da_1$$

Differentiating the above expression w.r.t. φ and adding equation (30) yield

$$\frac{\delta m}{3[2(r+\delta)+m\varphi]^3}\{\frac{[2(r+\delta)\delta-m\varphi\delta-2m^2\varphi^2]}{[\delta+m\varphi]^2}-\frac{2(r+\delta)}{[\delta+m]}\}+ \\ \frac{\delta m(r+\delta)}{3(\delta+m)[2(r+\delta)+m]}[\frac{1}{[2(r+\delta)+m(1-\varphi)]^2}-\frac{1}{[2(r+\delta)+m\varphi]^2}] \tag{31}$$

But for $\varphi = 0.5$, $\delta/[\delta + m\varphi]^2$ is less than $1/(m+\delta)$ and the last term is equal to zero. Hence, the derivative of the term inside the braces in equation (18) w.r.t. φ, it is always negative if it is evaluated at $\varphi = 0.5$ (the proof for $\partial\Gamma/\partial\varphi$ is similar).

Since $\dfrac{\delta s}{(r+\delta)}$ does not depend on θ, φ the analysis and the results in the case of firing taxes is the same.